Lecture 3
Uncertainty management in rule-based expert systems

- **Introduction, or what is uncertainty?**
- **Basic probability theory**
- **Bayesian reasoning**
- **Bias of the Bayesian method**
- **Certainty factors theory and evidential reasoning**
- **Summary**

Introduction, or what is uncertainty?

■ Information can be incomplete, inconsistent, uncertain, or all three. In other words, information is often unsuitable for solving a problem.

■ **Uncertainty** is defined as the lack of the exact knowledge that would enable us to reach a perfectly reliable conclusion. Classical logic permits only exact reasoning. It assumes that perfect knowledge always exists and the *law of the excluded middle* can always be applied:

IF A is true
THEN A is not false

IF A is false
THEN A is not true

Sources of uncertain knowledge

■ **Weak implications.** Domain experts and knowledge engineers have the painful task of establishing concrete correlations between IF (condition) and THEN (action) parts of the rules. Therefore, expert systems need to have the ability to handle vague associations, for example by accepting the degree of correlations as numerical certainty factors.

■ **Imprecise language.** Our natural language is ambiguous and imprecise. We describe facts with such terms as *often* and *sometimes*, *frequently* and *hardly ever*. As a result, it can be difficult to express knowledge in the precise IF-THEN form of production rules. However, if the meaning of the facts is quantified, it can be used in expert systems. In 1944, Ray Simpson asked 355 high school and college students to place 20 terms like *often* on a scale between 1 and 100. In 1968, Milton Hakel repeated this experiment.

Quantification of ambiguous and imprecise terms on a time-frequency scale

Ray Simpson (1944)		*Milton Hakel* (1968)	
Term	*Mean value*	*Term*	*Mean value*
Always	99	Always	100
Very often	88	Very often	87
Usually	85	Usually	79
Often	78	Often	74
Generally	78	Rather often	74
Frequently	73	Frequently	72
Rather often	65	Generally	72
About as often as not	50	About as often as not	50
Now and then	20	Now and then	34
Sometimes	20	Sometimes	29
Occasionally	20	Occasionally	28
Once in a while	15	Once in a while	22
Not often	13	Not often	16
Usually not	10	Usually not	16
Seldom	10	Seldom	9
Hardly ever	7	Hardly ever	8
Very seldom	6	Very seldom	7
Rarely	5	Rarely	5
Almost never	3	Almost never	2
Never	0	Never	0

- **Unknown data.** When the data is incomplete or missing, the only solution is to accept the value "unknown" and proceed to an approximate reasoning with this value.

- **Combining the views of different experts.** Large expert systems usually combine the knowledge and expertise of a number of experts. Unfortunately, experts often have contradictory opinions and produce conflicting rules. To resolve the conflict, the knowledge engineer has to attach a weight to each expert and then calculate the composite conclusion. But no systematic method exists to obtain these weights.

Basic probability theory

■ The concept of probability has a long history that goes back thousands of years when words like "probably", "likely", "maybe", "perhaps" and "possibly" were introduced into spoken languages. However, the mathematical theory of probability was formulated only in the 17th century.

■ The **probability** of an event is the proportion of cases in which the event occurs. Probability can also be defined as a *scientific measure of chance*.

■ Probability can be expressed mathematically as a numerical index with a range between zero (an absolute impossibility) to unity (an absolute certainty).

■ Most events have a probability index strictly between 0 and 1, which means that each event has *at least* two possible outcomes: favourable outcome or success, and unfavourable outcome or failure.

$$P(success) = \frac{the\ number\ of\ successes}{the\ number\ of\ possible\ outcomes}$$

$$P(failure) = \frac{the\ number\ of\ failures}{the\ number\ of\ possible\ outcomes}$$

■ If s is the number of times success can occur, and f is the number of times failure can occur, then

$$P(success) = p = \frac{s}{s+f}$$

$$P(failure) = q = \frac{f}{s+f}$$

and

$$p + q = 1$$

■ If we throw a coin, the probability of getting a head will be equal to the probability of getting a tail. In a single throw, $s = f = 1$, and therefore the probability of getting a head (or a tail) is 0.5.

Conditional probability

■ Let *A* be an event in the world and *B* be another event. Suppose that events *A* and *B* are not mutually exclusive, but occur conditionally on the occurrence of the other. The probability that event *A* will occur if event *B* occurs is called the **conditional probability**. Conditional probability is denoted mathematically as *p(A|B)* in which the vertical bar represents *GIVEN* and the complete probability expression is interpreted as *"Conditional probability of event A occurring given that event B has occurred"*.

$$p(A|B) = \frac{the\ number\ of\ times\ A\ and\ B\ can\ occur}{the\ number\ of\ times\ B\ can\ occur}$$

■ The number of times *A* and *B* can occur, or the probability that both *A* and *B* will occur, is called the **joint probability** of *A* and *B*. It is represented mathematically as $p(A \cap B)$. The number of ways *B* can occur is the probability of *B*, $p(B)$, and thus

$$p(A|B) = \frac{p(A \cap B)}{p(B)}$$

■ Similarly, the conditional probability of event *B* occurring given that event *A* has occurred equals

$$p(A|B) = \frac{p(A \cap B)}{p(B)}$$

Hence,

$$p(B \cap A) = p(B|A) \times p(A)$$

and

$$p(A \cap B) = p(B|A) \times p(A)$$

Substituting the last equation into the equation

$$p(A|B) = \frac{p(A \cap B)}{p(B)}$$

yields the **Bayesian rule**:

Bayesian rule

$$p(A|B) = \frac{p(B|A) \times p(A)}{p(B)}$$

where:

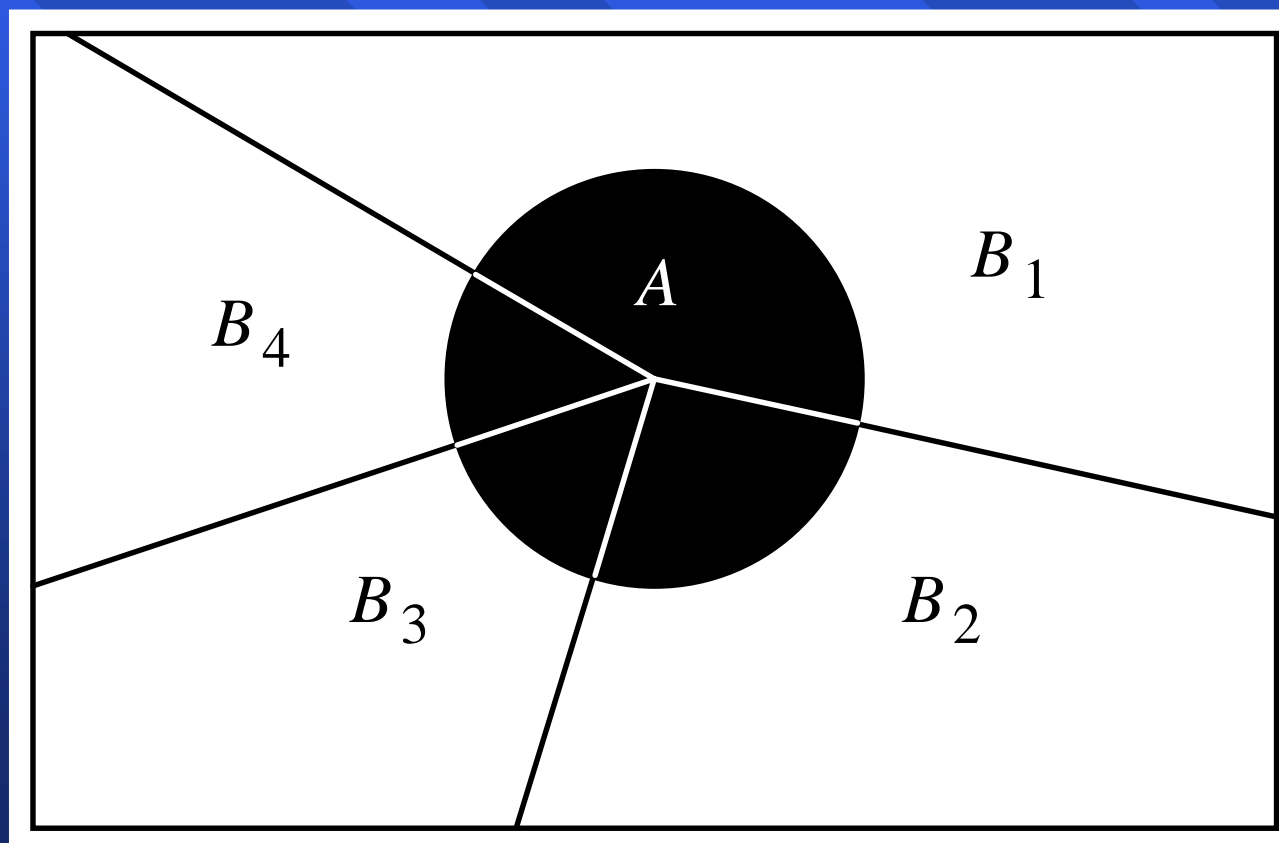$p(A|B)$ is the conditional probability that event A occurs given that event B has occurred;

$p(B|A)$ is the conditional probability of event B occurring given that event A has occurred;

$p(A)$ is the probability of event A occurring;

$p(B)$ is the probability of event B occurring.

The joint probability

$$\sum_{i=1}^{n} p(A \cap B_i) = \sum_{i=1}^{n} p(A|B_i) \times p(B_i)$$

If the occurrence of event *A* depends on only two mutually exclusive events, *B* and NOT *B*, we obtain:

$$p(A) = p(A|B) \times p(B) + p(A|\neg B) \times p(\neg B)$$

where $\neg$ is the logical function NOT.
Similarly,

$$p(B) = p(B|A) \times p(A) + p(B|\neg A) \times p(\neg A)$$

Substituting this equation into the Bayesian rule yields:

$$p(A|B) = \frac{p(B|A) \times p(A)}{p(B|A) \times p(A) + p(B|\neg A) \times p(\neg A)}$$

Bayesian reasoning

Suppose all rules in the knowledge base are represented in the following form:

IF H is true

THEN E is true {with probability p}

This rule implies that if event H occurs, then the probability that event E will occur is p.

In expert systems, H usually represents a hypothesis and E denotes evidence to support this hypothesis.

The Bayesian rule expressed in terms of hypotheses and evidence looks like this:

$$p(H|E) = \frac{p(E|H) \times p(H)}{p(E|H) \times p(H) + p(E|\neg H) \times p(\neg H)}$$

where:

$p(H)$ is the prior probability of hypothesis H being true;

$p(E|H)$ is the probability that hypothesis H being true will result in evidence E;

$p(\neg H)$ is the prior probability of hypothesis H being false;

$p(E|\neg H)$ is the probability of finding evidence E even when hypothesis H is false.

■ In expert systems, the probabilities required to solve a problem are provided by experts. An expert determines the **prior probabilities** for possible hypotheses $p(H)$ and $p(\neg H)$, and also the **conditional probabilities** for observing evidence E if hypothesis H is true, $p(E|H)$, and if hypothesis H is false, $p(E|\neg H)$.

■ Users provide information about the evidence observed and the expert system computes $p(H|E)$ for hypothesis H in light of the user-supplied evidence E. Probability $p(H|E)$ is called the **posterior probability** of hypothesis H upon observing evidence E.

- We can take into account both multiple hypotheses H_1, H_2,..., H_m and multiple evidences E_1, E_2,..., E_n. The hypotheses as well as the evidences must be mutually exclusive and exhaustive.

- Single evidence E and multiple hypotheses follow:

$$p(H_i|E) = \frac{p(E|H_i) \times p(H_i)}{\sum_{k=1}^{m} p(E|H_k) \times p(H_k)}$$

- Multiple evidences and multiple hypotheses follow:

$$p(H_i|E_1\,E_2\,...\,E_n) = \frac{p(E_1\,E_2\,...\,E_n|H_i) \times p(H_i)}{\sum_{k=1}^{m} p(E_1\,E_2\,...\,E_n|H_k) \times p(H_k)}$$

■ This requires to obtain the conditional probabilities of all possible combinations of evidences for all hypotheses, and thus places an enormous burden on the expert.

■ Therefore, in expert systems, conditional independence among different evidences assumed. Thus, instead of the unworkable *equation*, we attain:

$$p\left(H_i \middle| E_1\, E_2 \ldots E_n\right) = \frac{p\left(E_1 \middle| H_i\right) \times p\left(E_2 \middle| H_i\right) \times \ldots \times p\left(E_n \middle| H_i\right) \times p\left(H_i\right)}{\sum_{k=1}^{m} p\left(E_1 \middle| H_k\right) \times p\left(E_2 \middle| H_k\right) \times \ldots \times p\left(E_n \middle| H_k\right) \times p\left(H_k\right)}$$

Ranking potentially true hypotheses

Let us consider a simple example.

Suppose an expert, given three conditionally independent evidences E_1, E_2 and E_3, creates three mutually exclusive and exhaustive hypotheses H_1, H_2 and H_3, and provides prior probabilities for these hypotheses – $p(H_1)$, $p(H_2)$ and $p(H_3)$, respectively. The expert also determines the conditional probabilities of observing each evidence for all possible hypotheses.

The prior and conditional probabilities

Probability	Hypothesis		
	$i = 1$	$i = 2$	$i = 3$
$p(H_i)$	0.40	0.35	0.25
$p(E_1 \mid H_i)$	0.3	0.8	0.5
$p(E_2 \mid H_i)$	0.9	0.0	0.7
$p(E_3 \mid H_i)$	0.6	0.7	0.9

Assume that we first observe evidence E_3. The expert system computes the posterior probabilities for all hypotheses as

$$p(H_i|E_3) = \frac{p(E_3|H_i) \times p(H_i)}{\sum\limits_{k=1}^{3} p(E_3|H_k) \times p(H_k)}, \qquad i = 1, 2, 3$$

Thus,

$$p(H_1|E_3) = \frac{0.6 \cdot 0.40}{0.6 \cdot 0.40 + 0.7 \cdot 0.35 + 0.9 \cdot 0.25} = 0.34$$

$$p(H_2|E_3) = \frac{0.7 \cdot 0.35}{0.6 \cdot 0.40 + 0.7 \cdot 0.35 + 0.9 \cdot 0.25} = 0.34$$

$$p(H_3|E_3) = \frac{0.9 \cdot 0.25}{0.6 \cdot 0.40 + 0.7 \cdot 0.35 + 0.9 \cdot 0.25} = 0.32$$

After evidence E_3 is observed, belief in hypothesis H_2 increases and becomes equal to belief in hypothesis H_1. Belief in hypothesis H_3 also increases and even nearly reaches beliefs in hypotheses H_1 and H_2.

Suppose now that we observe evidence E_1. The posterior probabilities are calculated as

$$p(H_i|E_1E_3) = \frac{p(E_1|H_i) \times p(E_3|H_i) \times p(H_i)}{\sum\limits_{k=1}^{3} p(E_1|H_k) \times p(E_3|H_k) \times p(H_k)}, \qquad i = 1, 2, 3$$

Hence,

$$p(H_1|E_1E_3) = \frac{0.3 \cdot 0.6 \cdot 0.40}{0.3 \cdot 0.6 \cdot 0.40 + 0.8 \cdot 0.7 \cdot 0.35 + 0.5 \cdot 0.9 \cdot 0.25} = 0.19$$

$$p(H_2|E_1E_3) = \frac{0.8 \cdot 0.7 \cdot 0.35}{0.3 \cdot 0.6 \cdot 0.40 + 0.8 \cdot 0.7 \cdot 0.35 + 0.5 \cdot 0.9 \cdot 0.25} = 0.52$$

$$p(H_3|E_1E_3) = \frac{0.5 \cdot 0.9 \cdot 0.25}{0.3 \cdot 0.6 \cdot 0.40 + 0.8 \cdot 0.7 \cdot 0.35 + 0.5 \cdot 0.9 \cdot 0.25} = 0.29$$

Hypothesis H_2 has now become the most likely one.

After observing evidence E_2, the final posterior probabilities for all hypotheses are calculated:

$$p(H_i|E_1E_2E_3) = \frac{p(E_1|H_i) \times p(E_2|H_i) \times p(E_3|H_i) \times p(H_i)}{\sum_{k=1}^{3} p(E_1|H_k) \times p(E_2|H_k) \times p(E_3|H_k) \times p(H_k)}, \qquad i = 1, 2, 3$$

$$p(H_1|E_1E_2E_3) = \frac{0.3 \cdot 0.9 \cdot 0.6 \cdot 0.40}{0.3 \cdot 0.9 \cdot 0.6 \cdot 0.40 + 0.8 \cdot 0.0 \cdot 0.7 \cdot 0.35 + 0.5 \cdot 0.7 \cdot 0.9 \cdot 0.25} = 0.45$$

$$p(H_2|E_1E_2E_3) = \frac{0.8 \cdot 0.0 \cdot 0.7 \cdot 0.35}{0.3 \cdot 0.9 \cdot 0.6 \cdot 0.40 + 0.8 \cdot 0.0 \cdot 0.7 \cdot 0.35 + 0.5 \cdot 0.7 \cdot 0.9 \cdot 0.25} = 0$$

$$p(H_3|E_1E_2E_3) = \frac{0.5 \cdot 0.7 \cdot 0.9 \cdot 0.25}{0.3 \cdot 0.9 \cdot 0.6 \cdot 0.40 + 0.8 \cdot 0.0 \cdot 0.7 \cdot 0.35 + 0.5 \cdot 0.7 \cdot 0.9 \cdot 0.25} = 0.55$$

Although the initial ranking was H_1, H_2 and H_3, only hypotheses H_1 and H_3 remain under consideration after all evidences (E_1, E_2 and E_3) were observed.

Bias of the Bayesian method

■ The framework for Bayesian reasoning requires probability values as primary inputs. The assessment of these values usually involves human judgement. However, psychological research shows that humans either cannot elicit probability values consistent with the Bayesian rules.

■ This suggests that the conditional probabilities may be inconsistent with the prior probabilities given by the expert.

- Consider, for example, a car that does not start and makes odd noises when you press the starter. The conditional probability of the starter being faulty if the car makes odd noises may be expressed as:

  IF   the symptom is "odd noises"
  THEN the starter is bad {with probability 0.7}

- Consider, for example, a car that does not start and makes odd noises when you press the starter. The conditional probability of the starter being faulty if the car makes odd noises may be expressed as:

  p(starter is not bad|odd noises) =

  = p(starter is good|odd noises) = 1−0.7 = 0.3

- Therefore, we can obtain a companion rule that states
  IF   the symptom is "odd noises"
  THEN the starter is good {with probability 0.3}

- Domain experts do not deal with conditional probabilities and often deny the very existence of the **hidden implicit probability** (0.3 in our example).

- We would also use available statistical information and empirical studies to derive the following rules:

IF   the starter is bad
THEN the symptom is "odd noises" {probability 0.85}

IF   the starter is bad
THEN the symptom is not "odd noises" {probability 0.15}

- To use the Bayesian rule, we still need the **prior probability**, the probability that the starter is bad if the car does not start. Suppose, the expert supplies us the value of 5 per cent. Now we can apply the Bayesian rule to obtain:

$$p(starter\ is\ bad | odd\ noises) = \frac{0.85 \cdot 0.05}{0.85 \cdot 0.05 + 0.15 \cdot 0.95} = 0.23$$

- **The number obtained is significantly lower than the expert's estimate of 0.7 given at the beginning of this section.**

- The reason for the inconsistency is that the expert made different assumptions when assessing the conditional and prior probabilities.

Certainty factors theory and evidential reasoning

■ Certainty factors theory is a popular alternative to Bayesian reasoning.

■ A **certainty factor** (cf), a number to measure the expert's belief. The maximum value of the certainty factor is, say, +1.0 (definitely true) and the minimum −1.0 (definitely false). For example, if the expert states that some evidence is almost certainly true, a cf value of 0.8 would be assigned to this evidence.

Uncertain terms and their interpretation in MYCIN

Term	Certainty Factor
Definitely not	−1.0
Almost certainly not	−0.8
Probably not	−0.6
Maybe not	−0.4
Unknown	−0.2 to +0.2
Maybe	+0.4
Probably	+0.6
Almost certainly	+0.8
Definitely	+1.0

■ In expert systems with certainty factors, the knowledge base consists of a set of rules that have the following syntax:

IF   <evidence>
THEN <hypothesis> {*cf* }

where *cf* represents belief in hypothesis *H* given that evidence *E* has occurred.

■ The certainty factors theory is based on two functions: measure of belief *MB(H,E)*, and measure of disbelief *MD(H,E)*.

$$MB(H, E) = \begin{cases} 1 & \text{if } p(H) = 1 \\[2ex] \dfrac{max\left[p(H|E),\, p(H)\right] - p(H)}{max\left[1,\, 0\right] - p(H)} & \text{otherwise} \end{cases}$$

$$MD(H, E) = \begin{cases} 1 & \text{if } p(H) = 0 \\[2ex] \dfrac{min\left[p(H|E),\, p(H)\right] - p(H)}{min\left[1,\, 0\right] - p(H)} & \text{otherwise} \end{cases}$$

$p(H)$ is the prior probability of hypothesis H being true; $p(H|E)$ is the probability that hypothesis H is true given evidence E.

- The values of *MB*(*H*, *E*) and *MD*(*H*, *E*) range between 0 and 1. The strength of belief or disbelief in hypothesis *H* depends on the kind of evidence *E* observed. Some facts may increase the strength of belief, but some increase the strength of disbelief.

- **The total strength of belief or disbelief in a hypothesis:**

$$cf = \frac{MB(H,E) - MD(H,E)}{1 - min[MB(H,E), MD(H,E)]}$$

■ Example:

Consider a simple rule:

IF   *A* is *X*
THEN *B* is *Y*

An expert may not be absolutely certain that this rule holds. Also suppose it has been observed that in some cases, even when the IF part of the rule is satisfied and *object A* takes on *value X*, object *B* can acquire some different value *Z*.

IF   *A* is *X*
THEN *B* is *Y* {*cf* 0.7};
          B is *Z* {*cf* 0.2}

■ The certainty factor assigned by a rule is propagated through the reasoning chain. This involves establishing the net certainty of the rule consequent when the evidence in the rule antecedent is uncertain:

cf (*H*,*E*) = *cf* (*E*) × *cf*

For example,

IF   sky is clear
THEN the forecast is sunny {*cf* 0.8}

and the current certainty factor of *sky is clear* is 0.5, then

cf (*H*,*E*) = 0.5 · 0.8 = 0.4

This result can be interpreted as *"It may be sunny"*.

■ For conjunctive rules such as

IF   <evidence E_1>
$$\vdots$$
AND   <evidence E_n>
THEN <hypothesis H> {cf }

the certainty of hypothesis H, is established as follows:

$$cf\,(H, E_1 \cap E_2 \cap ... \cap E_n) = min\,[cf\,(E_1),\, cf\,(E_2),..., cf\,(E_n)] \times cf$$

For example,

IF   sky is clear
AND   the forecast is sunny
THEN   the action is 'wear sunglasses' {cf 0.8}

and the certainty of *sky is clear* is 0.9 and the certainty of the *forecast of sunny* is 0.7, then

$$cf\,(H, E_1 \cap E_2) = min\,[0.9,\,0.7] \cdot 0.8 = 0.7 \cdot 0.8 = 0.56$$

■ For disjunctive rules such as

IF   <evidence E_1>
                    ⋮
OR   <evidence E_n>
THEN <hypothesis H> $\{cf\}$

the certainty of hypothesis H, is established as follows:

$$cf\,(H, E_1 \cup E_2 \cup ... \cup E_n) = max\,[cf\,(E_1),\,cf\,(E_2),...,\,cf\,(E_n)] \times cf$$

For example,

IF   sky is overcast
OR   the forecast is rain
THEN the action is 'take an umbrella' $\{cf\,0.9\}$

and the certainty of *sky is overcast* is 0.6 and the certainty of the *forecast of rain* is 0.8, then

$$cf\,(H, E_1 \cup E_2) = max\,[0.6, 0.8] \cdot 0.9 = 0.8 \cdot 0.9 = 0.72$$

- When the same consequent is obtained as a result of the execution of two or more rules, the individual certainty factors of these rules must be merged to give a combined certainty factor for a hypothesis.

Suppose the knowledge base consists of the following rules:

Rule 1:	IF	*A* is *X*

	THEN	*C* is *Z* {*cf* 0.8}

Rule 2:	IF	*B* is *Y*

	THEN	*C* is *Z* {*cf* 0.6}

What certainty should be assigned to object *C* having value *Z* if both *Rule* 1 and *Rule* 2 are fired?

Common sense suggests that, if we have two pieces of evidence (*A* is *X* and *B* is *Y*) from different sources (*Rule* 1 and *Rule* 2) supporting the same hypothesis (*C* is *Z*), then the confidence in this hypothesis should increase and become stronger than if only one piece of evidence had been obtained.

To calculate a combined certainty factor we can use the following equation:

$$cf(cf_1, cf_2) = \begin{cases} cf_1 + cf_2 \times (1 - cf_1) & \text{if } cf_1 > 0 \text{ and } cf_2 > 0 \\[2em] \dfrac{cf_1 + cf_2}{1 - min\left[|cf_1|, |cf_1|\right]} & \text{if } cf_1 < 0 \text{ or } cf_2 < 0 \\[2em] cf_1 + cf_2 \times (1 + cf_1) & \text{if } cf_1 < 0 \text{ and } cf_2 < 0 \end{cases}$$

where:

cf_1 is the confidence in hypothesis H established by *Rule* 1;
cf_2 is the confidence in hypothesis H established by *Rule* 2;
$|cf_1|$ and $|cf_2|$ are absolute magnitudes of cf_1 and cf_2, respectively.

The certainty factors theory provides a *practical* alternative to Bayesian reasoning. The heuristic manner of combining certainty factors is different from the manner in which they would be combined if they were probabilities. The certainty theory is not "mathematically pure" but does mimic the thinking process of a human expert.

Comparison of Bayesian reasoning and certainty factors

■ Probability theory is the oldest and best-established technique to deal with inexact knowledge and random data. It works well in such areas as forecasting and planning, where statistical data is usually available and accurate probability statements can be made.

- However, in many areas of possible applications of expert systems, reliable statistical information is not available or we cannot assume the conditional independence of evidence. As a result, many researchers have found the Bayesian method unsuitable for their work. This dissatisfaction motivated the development of the certainty factors theory.

- Although the certainty factors approach lacks the mathematical correctness of the probability theory, it outperforms subjective Bayesian reasoning in such areas as diagnostics.

- Certainty factors are used in cases where the probabilities are not known or are too difficult or expensive to obtain. The evidential reasoning mechanism can manage incrementally acquired evidence, the conjunction and disjunction of hypotheses, as well as evidences with different degrees of belief.

- The certainty factors approach also provides better explanations of the control flow through a rule-based expert system.

- The Bayesian method is likely to be the most appropriate if reliable statistical data exists, the knowledge engineer is able to lead, and the expert is available for serious decision-analytical conversations.

- In the absence of any of the specified conditions, the Bayesian approach might be too arbitrary and even biased to produce meaningful results.

- The Bayesian belief propagation is of exponential complexity, and thus is impractical for large knowledge bases.